Modified Criterion of Hypothesis Testing for Signal Sensing in Cognitive Radio

Mohammed Alamgir, Michael Faulkner, Phillip Conder
Center for Telecommunications and Microelectronics
Victoria University, Melbourne, Australia
Email: mohammed.alamgir@live.vu.edu.au,
{mike.faulkner, phillip.conder}@vu.edu.au

Peter Smith
Department of Electrical and Computer Engineering
University of Canterbury
Christchurch, New Zealand.
Email:peter@elec.canterbury.ac.nz

***Abstract*—Signal detection problems are traditionally viewed as statistical hypothesis testing. In absence of the *a priori* probabilities, such as in radar, the Neyman-Pearson criterion is used where a certain false alarm probability is set, and the probability of detection is maximised. In signal sensing problems of cognitive radio, the main constraint is to avoid the interference with the primary user. Once this constraint is met, a cognitive radio can maximise its own chance of finding an empty spectrum. In this paper we emphasise this view of the signal sensing problem and modify the criterion such that a maximum miss-detection rate is specified. We have reformulated the energy detector showing that the sensing results have more meaningful explanations under the modified criterion. The effects of measurement errors are also considered.**

***Index Terms*—Cognitive Radio, signal detection, Neyman-Pearson, hypothesis testing, energy detector.**

I. Introduction

In wireless communication the demands for greater speed, more reliability and wider coverage are ever increasing and so is the need for new spectra. But there is very limited amount of unallocated spectrum available in the usable bands of $<$ 3GHz [1]. At the same time, the utilisation of the allocated spectra is very low as found in [2] and other studies. Cognitive radio (CR) is being thought as a communication paradigm for the future where a spectrum that has been licensed to a primary user can be accessed by a secondary user when idle [3]. The secondary user must not use a spectrum which is already in use, and must free up the spectrum when the primary user begins using it. The regulators, such as the FCC in the US, ACMA in Australia are currently studying the implications of allowing such spectrum sharing [4].

The problem of signal detection has been studied well and applied to radar applications for detecting targets [5][6]. Two types of errors are possible in a simple detection: false alarm and miss-detection. A false alarm occurs when the detector reports the presence of a target while it is absent in reality. A miss-detection is a failure of the detector when a target is present. A false alarm causes an action, for example, in defense, a missile is fired wasting resources. On the other hand, a miss-detection results in loss or causalities. Total avoidance of any loss is always desirable, but the miss-detection probability can not be made zero for a finite detection period. For a given detection period the miss-detection probability can be reduced by allowing increased false alarms, which again costs more resources. Since both of the errors can not be minimised at the same time, a certain false alarm rate is permitted depending on the operational budget, and the probability of miss-detection is minimised.

In CR, a secondary user finds an empty spectrum by sensing the media for possible signals from primary users. A false alarm in such case only means that the secondary user would not transmit in an empty spectrum during an idle period. This would only affect the throughput of the CR, but would not have any impact on the primary user. However, if the secondary user misses a primary signal and starts transmission, then it would cause interference with the primary user. Should future spectrum licenses allow the operation of cognitive radio, we hypothesise that any potential primary users would seek assurances on the maximum level of anticipated interference which the regulators would need to specify in the licensing conditions.

Specifying a false alarm rate helps a secondary user maintain its own throughput; it does not ensure anything about the interference with the primary user. Thus, the constraint of operation of the CR is not met. Recent CR studies seem to miss this point, and continue to report sensing results by specifying false alarm rates [7]. To guarantee that the primary user is not interfered with more than a certain fraction of time, the secondary user needs to have its miss-detection rate specified instead of the false alarm rate. In this paper, we reframe the signal detection problem, and argue that the new view captures the constraints on CR more naturally than the view of the radar target detection problem.

This rest of the document is organised as follows: The detection model and criterion are given in Section II, Section III presents the energy detector under the modified criterion. Effects of measurement errors are given in Section IV, results are shown and discussed in Section V followed by concluding remarks in Section VI.

II. Detection Model and Decision Criterion

Signal detection is usually viewed as a decision theoretic problem, and commonly modelled to have four components: a source that generates outputs (called hypotheses), a probabilistic mechanism, an observation space and a decision rule

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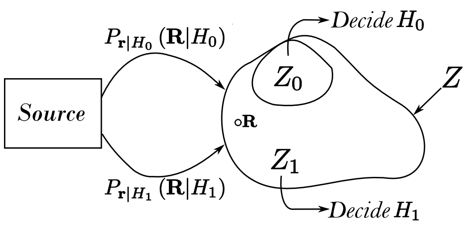

Fig. 1. Decision regions.

(Fig. 1) [8]. The probabilistic mechanism transforms each output of the source into a point in the observation space which is the only thing accessible to a tester.

Let us consider an observation vector $\mathbf{r}$ of N values generated by the probability density functions (PDF) $p_{\mathbf{r}|H_0}(\mathbf{R}|H_0)$ and $p_{\mathbf{r}|H_1}(\mathbf{R}|H_1)$, where $\mathbf{R}$ is a random variable. Here H_0 and H_1 are the hypothesis that the primary signal is absent and present, respectively. Based on $\mathbf{r}$, the test must choose either H_0 or H_1 corresponding to the decision region Z_0 and Z_1. So, for the whole observation space $Z = Z_0 \cup Z_1$. There are four possibilities in making the decision:

1) H_0 is true; H_0 is chosen;
2) H_0 is true; H_1 is chosen;
3) H_1 is true; H_1 is chosen;
4) H_1 is true; H_0 is chosen;

where the first and third are correct, the second and fourth are wrong. We call the first case an *acquisition* since this is the situation where a cognitive radio finds a spectrum empty. The rest of the cases are respectively called *false alarm*, *detection* and *miss-detection*. We have the probability of acquisition,

$$P_A = \int_{Z_0} p_{\mathbf{r}|H_0}(\mathbf{R}|H_0)d\mathbf{R}, \tag{1}$$

the probability of false alarm,

$$P_F = \int_{Z_1} p_{\mathbf{r}|H_0}(\mathbf{R}|H_0)d\mathbf{R}, \tag{2}$$

the probability of detection,

$$P_D = \int_{Z_1} p_{\mathbf{r}|H_1}(\mathbf{R}|H_1)d\mathbf{R}, \tag{3}$$

and the probability of miss-detection,

$$P_M = \int_{Z_0} p_{\mathbf{r}|H_1}(\mathbf{R}|H_1)d\mathbf{R} \tag{4}$$

and the *a priori* probabilities $P_0 = Pr(H_0)$ and $P_1 = Pr(H_1)$. Let us assign some cost C_{ij} to choosing H_i whereas H_j was true. Logically, the cost of wrong decisions are higher than the correct ones, so $C_{10} > C_{00}$ and $C_{01} > C_{11}$. The total cost of making a decision

$$\begin{aligned} TC =& C_{00}P_0Pr(\text{choose } H_0|H_0 \text{ is true})+ \\ & C_{10}P_0Pr(\text{choose } H_1|H_0 \text{ is true})+ \\ & C_{11}P_1Pr(\text{choose } H_1|H_1 \text{ is true})+ \\ & C_{01}P_1Pr(\text{choose } H_0|H_1 \text{ is true}) \end{aligned} \tag{5}$$

or

$$\begin{aligned} TC = C_{00}P_0P_A + C_{10}P_0P_F+ \\ C_{11}P_1P_D + C_{01}P_1P_M \end{aligned} \tag{6}$$

which after using the fact $P_A + P_F = 1$ and $P_D + P_M = 1$ becomes,

$$\begin{aligned} TC = P_0C_{00} + P_1C_{11} + P_1(C_{01} - C_{11})P_M+ \\ P_0(C_{10} - C_{00})(1 - P_F). \end{aligned} \tag{7}$$

In the Neyman-Pearson criterion [8], P_F is subjected to a constraint $P_F = \alpha' \leq \alpha$ for a given α, and then P_D is maximised. However, it is P_M that determines the interference perceived by the primary user. Therefore, the criterion of subjecting P_F to a certain value does not meet any interference requirement. To meet the interference obligation, we subject $P_M = \alpha' \leq \alpha$ and design a test that maximises P_A (or equivalently minimises P_F). For this modified criterion, we need to minimise

$$\begin{aligned} F =& P_F + \lambda' \left[P_M - \alpha'\right] \\ =& \int_{Z_1} p_{\mathbf{r}|H_0}(\mathbf{R}|H_0)d\mathbf{R}+ \\ & \lambda' \left[\int_{Z_0} p_{\mathbf{r}|H_1}(\mathbf{R}|H_1)d\mathbf{R} - \alpha'\right]. \end{aligned} \tag{8}$$

Obviously, if $P_M = \alpha'$, then minimising F minimises P_F. Since, $Z_0 = Z - Z_1$ (where $-$ means the set difference operation), from (8) we have

$$\begin{aligned} F =& \int_{Z_1} p_{\mathbf{r}|H_0}(\mathbf{R}|H_0)d\mathbf{R} + \lambda' \left[\int_{Z-Z_1} p_{\mathbf{r}|H_1}(\mathbf{R}|H_1)d\mathbf{R} - \alpha'\right] \\ =& \lambda'(1-\alpha') + \int_{Z_1} \left[\underbrace{p_{\mathbf{r}|H_0}(\mathbf{R}|H_0)}_{A} - \underbrace{\lambda' p_{\mathbf{r}|H_1}(\mathbf{R}|H_1)}_{B}\right] d\mathbf{R} \end{aligned} \tag{9}$$

To minimise F, we assign a point $\mathbf{r}$ to Z_1 that results in $B > A$. This is because we would like to make the term $A - B$ negative for a positive λ'. Points that make $A > B$ go to Z_0, and the points for $A = B$ can arbitrarily go to either Z_1 or Z_0. Therefore, we choose H_1 when

$$p_{\mathbf{r}|H_0}(\mathbf{R}|H_0) - \lambda' p_{\mathbf{r}|H_1}(\mathbf{R}|H_1) < 0 \tag{10}$$

or,

$$\frac{p_{\mathbf{r}|H_1}(\mathbf{R}|H_1)}{p_{\mathbf{r}|H_0}(\mathbf{R}|H_0)} > \lambda \tag{11}$$

where we let $\lambda = \frac{1}{\lambda'}$ since it is the sign of λ' that determines the minimisation. This is the same likelihood ratio that one obtains for the Neyman-Pearson criterion.

The specification of P_M instead of P_F for meeting the interference obligation can also be explained graphically. Fig. 2 shows the PDFs of H_0 and H_1 and the different error regions. The threshold λ can be specified from either of the shaded regions representing P_F and P_M. However, for guarantying

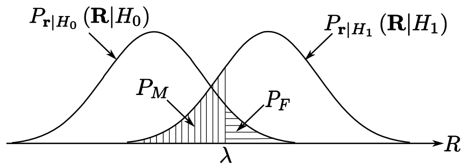

Fig. 2. PDF of H_0 and H_1 and the error areas.

a specified maximum tolerable level of interference, we must have the P_M specified.

In radar applications, the area P_F is usually taken as given because the PDF of H_0 is commonly known. The other reason is that a radar does not care anything about interfering with the target. For signal sensing problem in CR the primary user can not be ignored, thus the area P_M should be specified to limit the interference. This, will require the PDF of H_1.

III. Energy Detector

Given N samples x_i and the hypotheses

$$H_0 : x_i = n_i \qquad (12)$$
$$H_1 : x_i = s_i + n_i$$

where n_i is the noise, s_i is the primary signal, and $i = 1, 2, \cdots, N$. If the samples are Gaussian and independently and identically distributed (IID), the energy detector (ED) chooses H_1 when

$$T = \sum_{i=1}^{N} V_i \geq \lambda \qquad (13)$$

where we let $V_i = x_i^2$, and λ is the decision threshold to be determined.

If the variance of the noise σ_n^2, and that of the primary signal σ_s^2 are known, the PDF of the decision (13) has χ^2 distribution [9]. For reliable detection the sensing time needs to be long, hence the number of samples N is assumed large enough to approximate the PDFs with normal distributions [10].

Under H_0, the mean, $E[V_i] = \sigma_n^2$ and the variance, $E[(V_i - E[V_i])^2] = 2\sigma_n^4$ where we applied the fact that cumulant of order higher than two is zero for Gaussian variables. Since V_is are assumed IID, the mean and variance of T are $\mu_0 = N\sigma_n^2$ and $\sigma_0^2 = N\sigma_n^4$, respectively. Similarly, under H_1, T is normally distributed with mean, $\mu_1 = N(\sigma_n^2 + \sigma_s^2)$ and variance, $\sigma_1^2 = N(\sigma_n^2 + \sigma_s^2)^2$. Explicitly

$$PDF_{H_0}(t) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left[-\frac{(t-\mu_0)^2}{2\sigma_0^2}\right]$$
$$PDF_{H_1}(t) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left[-\frac{(t-\mu_1)^2}{2\sigma_1^2}\right] \qquad (14)$$

where t is the real variable for random variable T.

The probability of miss-detection, P_M, and the probability of acquisition, P_A, are given by

$$P_M = R\left(\frac{\lambda - \mu_1}{\sigma_1}\right) \qquad (15)$$

and

$$P_A = R\left(\frac{\lambda - \mu_0}{\sigma_0}\right) \qquad (16)$$

where

$$R(l) = \frac{1}{\sqrt{2\pi}} \int_0^l e^{-y^2/2} dy. \qquad (17)$$

In traditional energy detection, the threshold λ is calculated for a specified false alarm rate. Let us call this the Original Criterion (OC). For CR, if we choose P_{Mmax} as the maximum miss-detection rate that meets the interference obligation to the primary user, then under this Modified Criterion (MC), from (15) the decision threshold is

$$\hat{\lambda} = \mu_1 + \sigma_1 R^{-1}(P_{Mmax}). \qquad (18)$$

Under this criterion, for setting the proper threshold the PDF of H_1 is needed which in turn requires the estimation of $\sigma_n^2 + \sigma_s^2$. With this $\hat{\lambda}$, the acquisition probability can be found from (16).

IV. Effects of Measurement Error

Under the original criterion, the decision threshold is proportional to the noise power. If the noise power is over-estimated, the resultant false alarm rate is lower than the desired, and so is the probability of detection. In the case of under-estimation, the resultant probability of false alarm and detection, both go high [6]. For a CR using the OC, an over-estimation is prohibitive, since it would mean lower probability of detection thus risk of exerting more interference. It can however, underestimate the noise at the expense of higher false alarms, thereby reducing its own throughput.

Similarly, under the modified criterion, an overestimated σ_{sn}^2 would result in $P_M > P_{Mmax}$ which means more interference than the specified level. If σ_{sn}^2 is underestimated, then $P_M < P_{Mmax}$ which is good for the primary user; but for the cognitive radio now the probability of acquisition would be low.

Let us consider the measured power, $\sigma^2 = \epsilon\sigma^2$, where the error ϵ (expressed in dB) is a uniform random variable in an interval $[-B, +B]$. For noise, the nominal values of B has been found around 1 to 2 dB [11]. When measuring combined signal and noise power, we can expect similar error.

V. Results and Discussion

We evaluate the criteria by simulating an energy detector. A nominal noise power is set, and the corresponding signal power is calculated for a few signal to noise ratio (SNR) values.

In the first set of experiment we evaluate the effect of measurement uncertainty. Under the OC, the decision thresholds are calculated for a range of P_F values using only the noise power. The false alarm events are counted after running the test on only noise. Under the MC, the decision thresholds are calculated using (18) for a range of P_M values using the combined signal and noise power. The test is run on combined signal and noise, and the event of miss-detection is counted.

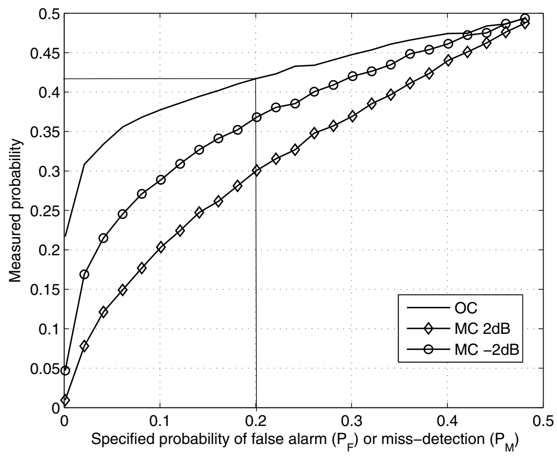

Fig. 3. Measured probability of miss and false alarm. SNR values are -2, 0 and 2 dB. Uncertainty, $B = 1$dB.

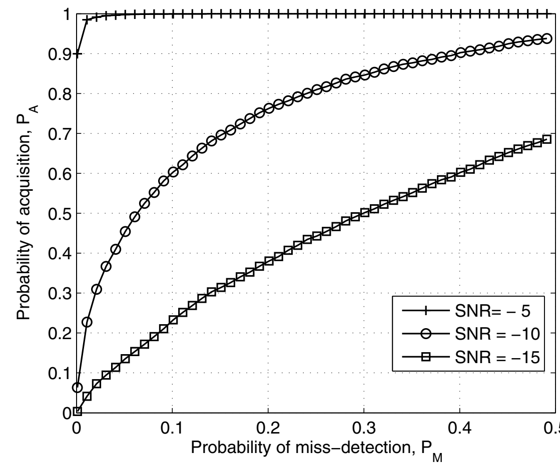

Fig. 4. Sensing performance of the modified criterion. SNR values are -5, -10 and -15dB. Uncertainty, $B = 1$dB, N=500.

Fig. 3 shows how a 1dB uncertainty in the noise measurement affects the false alarm (for OC) and miss-detection rates (for MC). For example, a specified false alarm rate of 0.2 leads to an actual observed false alarm rate of about 0.41. For MC, the miss-detection rate degrades less because the combined power is always greater than the noise power alone.

The SNR=2dB trace in Fig. 2 implies that the measured miss-detection probabilities are close to the specified values for a strong primary signal. Under such condition, a secondary user using the MC will be able to guarantee the interference obligation. The OC can not be used in such scenario since no matter how small or large the specified false alarm rate is, it does not guarantee any kind of immunity to the primary user. If the primary signal is weak, then the MC is no worse than the OC.

The MC needs to know the PDF of H_1, which in present case is the PDF of the energy of the primary signal. This can be learnt in a static environment, but could be difficult in a mobile channel where the average signal power changes. Therefore, MC should be applied to tests where the PDF of H_1 is readily available. It is also possible to design sensing methods where the PDF of H_1 is more robust to changes in the signal level [12].

The sensing performance for the MC can be shown by plotting probability of acquisition against specified miss-detection rates. The average SNR and number of samples can be chosen as the parameter. For example, Fig. 4 shows the acquisition performance of an energy detector under the MC for different average SNRs. This plot is more appealing than a P_D versus P_F plot since the relations between the pair of probabilities (P_D,P_A) and (P_F,P_M) are not always obvious.

VI. Conclusion

We emphasised that it is the miss-detection rate that determines the level of interference a secondary user might have with a primary user. It was hypothesised that the regulators would specify a maximum miss-detection rate for cognitive radio operators to guarantee the rights of the primary users. With this view, we have proposed a modification to the common signal sensing criterion that better suits the need of cognitive radio. The modified criterion ensures that a secondary user does not exceed a specified maximum interference to the primary users. The energy detector has been reformulated as a demonstration of the modified criterion. The criterion is applicable to any sensing method as long as the PDF of H_1 is known.

References

[1] FCC, “United States Frequency Allocations- The Radio Spectrum,” Available online: http://www.ntia.doc.gov/osmhome/allochrt.pdf
[2] D. Cabric, S. M. Mishra and R. W. Brodersen, “Implementation Issues in Spectrum Sensing for Cognitive Radios,” *38th Annual Asilomar Conference on Signals,* Systems and Computers, November 2004.
[3] J. Mitola and G.Q. Maguire, “Cognitive radios: making software radios more personal,” *IEEE Personal Communications,* vol. 6, no. 4, pp. 13-18, 1999.
[4] FCC, “Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies, notice of proposed rule making and order,” in *FCC 03-322*, December 2003.
[5] W. A. Gardner, “Signal Interception: A Unifying Theoretical Framework for Feature Detection,” *IEEE Transaction on Communications,* vol. 36, no. 8, pp. 897-906, August 1988.
[6] A. Sonnenschein and P. M. Fishman, “Radiometric Detection of Spread-Spectrum Signals in Noise Uncertain Power,” *IEEE Trans. on Aerospace and Elec. Systems,* vol. 28, no. 3, July 1992.
[7] S. Shellhammer, G. Chouinard, M. Muterspaugh, and M. Ghosh, ‘Spectrum sensing Simulation model,’ available online: http://grouper.ieee.org/groups/802/22/, July 2006.
[8] H. L. Van Trees, *Detection, Estimation, and Modulation Theory- Part I. Detection, Estimation, and Linear Modulation Theory,* John Wiely, New York, 2001.
[9] S. M. Kay, “Fundamentals of Statistical Signal Processing, Volume 2: Detection Theory,” *PTR Prentice Hall,* Upper Saddle River, New Jersey, 1998.
[10] H. Urkowitz, “Energy Detection of Unknown Deterministic Signals,” *Proc. of the IEEE,* vol. 55, pp. 523-531, April 1967.
[11] IEEE 802.22 Wireless RAN, Functional requirements for the 802.22 WRAN standard, *IEEE 802.22- 05/0007r46,* October 2005.
[12] M. Alamgir, M. Falkner and P. Conder, ‘Signal Sensing for Cognitive Radios Using Synthetic Multipath,’ in Proc. of Int. Conf. on Signal Proc. and Comm. Systems, ICSPCS’08, Gold Coast, Australia, December 2008.